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CALCULATION OF (e^+, e^-)-PAIR CONTRIBUTION TO THE SPECTRA OF ELECTRONS SCATTERED BY NUCLEI

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The electronuclear measurements always include the background from the (e^+, e^-)-pair photoproduction reaction in the target. The attempt to take into account this background through its computation by usual methods with applying the computation program GEANT-3 has appeared impractical, since it would take several years for the PC to operate. The present paper describes the technique, which permits one to reduce the time required for the PC computation of the background to a few hours. The developed technique has been verified by carrying out computations of the background from the (e^+, e^-)-pairs for the conditions, at which this background was measured at Saclay. The comparison between the calculated data and the experimental results has shown their agreement.
KEYWORDS: electronuclear measurements, target, pair photoproduction, background, simulation

РОЗРАХУНОК ВКЛАДУ (e^+, e^-)-ПАР В СПЕКРИ ЕЛЕКТРОНІВ, РОЗСІЯНИХ НА ЯДРАХ
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В даних електроядерних вимірювань присутній фон від фотонародження (e^+, e^-)-пар в мішені. Спроба врахувати цей фон шляхом його розрахунку показала, що при звичайній методології використання обчислювальної програми GEANT-3 такий розрахунок неможливий, тому що необхідний для нього час роботи ПК складає порядок кількох років. В даній роботі розглянута методика, яка дозволяє зменшити час роботи ПК, що потрібен для розрахунку цього фону, до кількох годин. Перевіркою знайденої методики стали виконані за її допомогою розрахунки (e^+, e^-)-фону для умов, при яких цей фон був виміряний в лабораторії Saclay. Порівняння результатів розрахунку та експерименту показало їх узгодження.
KEYWORDS: електроядерні вимірювання, мішень, фотонародження пар, фон, моделювання

РАСЧЁТ ВКЛАДА (e^+, e^-)-ПАР В СПЕКТРЫ ЭЛЕКТРОНОВ, РАССЕЯННЫХ НА ЯДРАХ
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В электроядерных измерениях всегда присутствует фон от фоторождения в мишени (e^+, e^-)-пар. Попытка учесть этот фон путём его расчёта показала, что при обычной методологии применения вычислительной программы GEANT-3 такой расчёт невозможен, так как необходимое для него время работы ПК порядка нескольких лет. В настоящей работе разработана методика, которая позволяет уменьшить требующееся для расчёта этого фона время работы ПК до нескольких часов. Проверкой найденной методики были выполненные с её помощью расчёты фона от (e^+, e^-)-пар для условий, при которых этот фон был измерен в лаборатории Saclay. Сравнение результатов расчёта и эксперимента показало их согласие.
KEYWORDS: электроядерные измерения, мишень, фоторождение пар, фон, моделирование

The electron-nucleus scattering spectrum comprises the contribution of electrons from the (e^+, e^-)-pair photoproduction reaction in the target. Its measurement procedure includes the magnetic polarity reversal of the spectrometer, and the measurement of the positron spectrum, which is similar to the electron spectrum from (e^+, e^-)-pairs. However, these measurements may take a substantial part of the time of experiment, i.e., costly running time of the accelerator. Therefore, it is reasonable to replace the measurements of the background by its computation.

The spectrum of positrons escaping from the electron-irradiated target can be calculated by the numerical simulation methods, using the computation program GEANT-3 [1] for the purpose. However, the estimation of time required for this computation has shown that it would take no less than 10^4 stream days of the PC. Hence, should the need arise to calculate the background of (e^+, e^-)-pairs, then for the solution of the problem it would be necessary to change the computation procedure in such a way as to increase the computing speed by several orders of magnitude. Precisely this optimization of computation is the aim of the present work.

(e^+, e^-)-PAIR PRODUCTION IN THE TARGET BOMBARDED BY ELECTRONS

The (e^+, e^-)-pair production in the electron-irradiated target is the result of two nuclear reactions occurring one after another, viz., bremsstrahlung photon emission by incident electron ($e^- \rightarrow (e^{-\prime}, \gamma)$ reaction), and electron-positron pair production by this photon ($\gamma \rightarrow (e^+, e^-)$ reaction). The calculation of the bremsstrahlung photon spectrum is one of the procedures, which is executed by the GEANT-3 program most often. As regards the (e^+, e^-)-pair photoproduction, the cross-section for this reaction can be calculated at certain conditions with the use of analytical formulas from [2].

The comparison between the data calculated by those formulas and the results of computations by the GEANT-3 program is given in Fig. 1. It is obvious from the figure that at positron energies E', which correspond to the conditions of application of the equations of [2], the data calculated by those equations are in agreement with the CEANT-3 computations. Let electrons incident on the target have equal energies E_0. The bremsstrahlung caused by these electrons presents a continuous spectrum with photon energies $E\gamma$ ranging from 0 up to $E_0 - m_e$, where m_e is the electron mass. We write down the yield of bremsstrahlung photons of energy $E\gamma$ in terms of the differential cross-section $d\sigma_{e-,\gamma}(E_0,E_\gamma)/dE_\gamma$ for the reaction $e^- \rightarrow (e^{-\prime}, \gamma)$

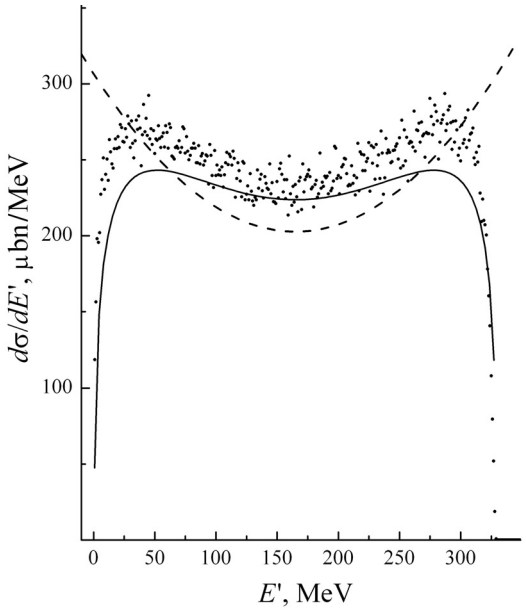

Fig. 1. Differential cross-section for (e^+, e^-)-pair photoproduction by ^{6}Li versus positron energy E'.
The photon energy is $E_\gamma = 330$ MeV. The points show the GEANT-3 computations, the dashed line is the complete screening case, (eq. (33.20), [2]), the solid line – absence of screening (eq. (33.19), [2]).

$$N_\gamma\left(E_0,E_\gamma\right)=\frac{d\sigma_{e-,\gamma}\left(E_0,E_\gamma\right)}{dE_\gamma}n_{e-}n_n\,, \qquad (1)$$

where n_{e-} is the number of electrons arrived at the target, n_n is the target thickness represented as the number of nuclei per cm^2. The yield of positrons of energy E' from the (e^+, e^-)-pair photoproduction reaction is represented similarly by

$$N_{e+}\left(E_\gamma,E'\right)=\frac{d\sigma_{\gamma,e+}\left(E_\gamma,E'\right)}{dE'}n_\gamma n_n\,. \qquad (2)$$

Here $d\sigma_{\gamma,e+}(E_\gamma,E')/dE'$ is the differential cross-section for the reaction $\gamma \rightarrow (e^+, e^-)$; n_γ is the number of photons that hit the target and have the energy E_γ. The variable n_γ is the yield $N_\gamma(E_0,E_\gamma)$, which is determined by (1). Substituting (1) for the variable n_γ in (2), and then integrating (2) over the whole spectrum of bremsstrahlung photons, we obtain the positron spectrum originating from the electrons of energy E_0:

$$N'_{e+}\left(E_0,E'\right)=\int_0^{E_0-m_e}\frac{d\sigma_{\gamma,e+}\left(E_\gamma,E'\right)}{dE'}\times\frac{d\sigma_{e-,\gamma}\left(E_0,E_\gamma\right)}{dE_\gamma}n_{e-}n_n^2\,dE_\gamma\,. \qquad (3)$$

It is not difficult to see that (3) describes the case, where the bremsstrahlung photons are emitted in the target of thickness n_n, and then, in the other target of the same thickness these photons give rise to the production of (e^+, e^-)-pairs. What actually happens is that the bremsstrahlung photon emission takes place over the whole electron trajectory in the target substance. We divide the target thickness into K identical layers. In a thin target, the yield of bremsstrahlung from each layer is the same, and this yield from an arbitrary i-th layer can be written as

$$N_{\gamma,i}\left(E_0,E_\gamma\right)=N_\gamma\left(E_0,E_\gamma\right)\big/K\,, \qquad (4)$$

where $N_\gamma(E_0,E_\gamma)$ is determined by (1). However, the probability of inducing the (e^+, e^-)-pair photoproduction reaction depends on the number of layers between the point of photon emission and the back surface of the target. For example, for the photon emitted near the front surface of the target, the probability of occurrence of the $\gamma \rightarrow (e^+, e^-)$ reaction is maximum, whereas the photon, emitted near the back surface of the target, immediately leaves the target substance, and thus cannot induce any reaction. We numerate the layers starting from the front target surface downstream. Then the number of (e^+, e^-)-pairs produced by the photons from the i-th layer will be given by

$$N_{e+,i}\left(E_0,E'\right)=N'_{e+}\left(E_0,E'\right)\times\left(1-i/K\right)\big/K\,. \qquad (5)$$

Let us make summation of equation (5) from $i = 1$ up to $i = K - 1$, and let K tend to infinity. As a result, we obtain the expression for the positron yield from all bremsstrahlung photons produced in the target exposed to electrons of energy E_0

$$N_{e+}\left(E_0,E'\right)=N'_{e+}\left(E_0,E'\right)\times 1/2\,, \qquad (6)$$

where $N'_{e+}(E_0,E')$ is determined by (3).

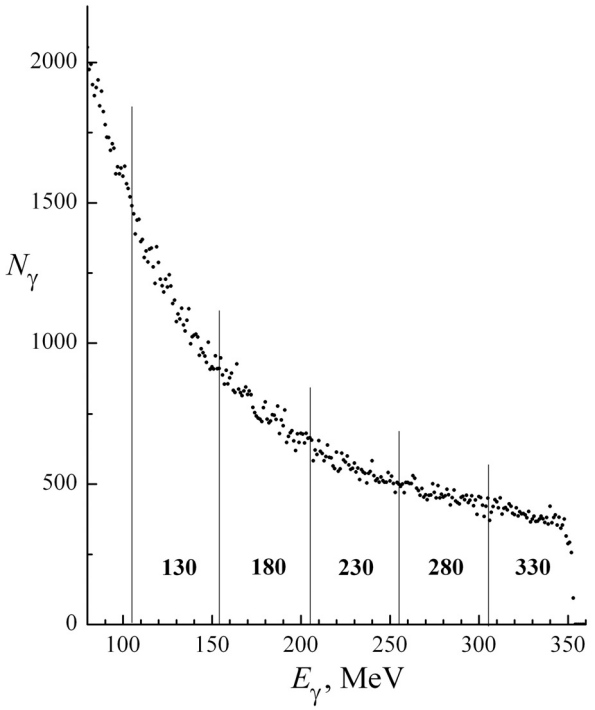

Fig. 2. Photon bremsstrahlung spectrum calculated by GEANT-3.
Vertical lines show energy intervals, ΔE_γ = 50 MeV in width, bold-faced figures show average energies $E_{\gamma,L}$ for the intervals.

This expression can be represented as a structure consisting of two hypothetic targets. The first target A of thickness $½n_n$ generates bremsstrahlung photons, and in the second target B of thickness n_n the photons from the first target give rise to (e^+, e^-)-pairs. We call this representation of the target as the "double target" (DT) model.

The computation of the bremsstrahlung photon spectrum does not take much time of PC operation. Fig. 2 exemplifies the result of the computation of this spectrum. We divide the spectrum into several similar energy intervals of width ΔE_γ, the average energy in each interval being $E_{\gamma,L}$. Let us consider the spectrum under discussion as a set of several groups of monoenergetic photons, each group having the energy $E_{\gamma,L}$ with the number of photons $N_{\gamma,L}$, which have this energy. The $N_{\gamma,L}$ value is determined by the area of the spectral region lying within the photon energy range $E_{\gamma,L} \pm \Delta E_\gamma/2$. The number of photons per bremsstrahlung photon from the L-th interval is given by $\eta_L = n_{e^-}/N_{\gamma,L}$.

We apply GEANT-3 for computing the passage of photons of energy $E_{\gamma,L}$ through the target B (the DT model). If the number of photons is sufficient, then, as a result, for the photon group of energy $E_{\gamma,L}$ we obtain the positron yield in the form of the spectrum $N_{e+,L}(E_0, E')$. After computations for all the energies of monoenergetic photons, it is necessary to sum the positron spectra corresponding to these photons. However, for each positron spectrum of that sum we need to have the weighting factor that would normalize the spectrum to the number of electrons, which should fall on the target in order for the spectrum to be obtained. In view of the normalization, the sum representing the positron spectrum may be written as

$$N_{e^+}(E_0, E') = \sum_L \frac{N_{e^+,L}(E_0, E')}{n_\gamma \times \eta_L}. \tag{7}$$

THE SPECTRUM OF POSITRONS COMING TO THE SPECTROMETER

Experimental positron spectra measured from ^{208}Pb at E_0 = 354 and 645 MeV have been given in paper [3]. For the purposes of comparison between the computed results and the experimental positron yield, we have used in our computations the spectrum measurement conditions [3], i.e., the target thickness in units of radiation length $t_0 = 1.6\times10^{-2}$, the exit angle $\theta = 60°$, acceptance angle of the spectrometer $\Delta\Omega_0 = 7\times10^{-3}$ sr. Besides, the positron yield from the ^{208}Pb target was calculated at E_0 = 354 MeV, $\theta = 20°$, and from the ^{6}Li target of thickness $t_0 = 0.3\times10^{-2}$ rad. length at E_0 = 260 MeV, $\theta = 15°$ and 35°.

Further, the computations are exemplified by the ^{208}Pb target and E_0 = 354 MeV.

If we assume that the collimator of the spectrometer is a square in section, then using the $\Delta\Omega_0$ value it is not difficult to calculate the acceptance of the spectrometer in the scattering angle: $\Delta\theta = 4.8°$. The positron yield is axially symmetric. Therefore, to speed up the computation, we replace the acceptance of spectrometer $\Delta\Omega_0$ by the solid angle included between the cones with openings $2\theta_1$ and $2\theta_2$, where $\theta_1 = \theta - \Delta\theta/2$ and $\theta_2 = \theta + \Delta\theta/2$. This solid angle is $\Delta\Omega = 2\pi(\cos\theta_1 - \cos\theta_2)$ sr. If on a certain interval $\Delta\theta' > \Delta\theta$, the N_{e+} yield is assumed to be a linear function of θ, then in the computations, $\Delta\Omega$ may be replaced by $\Delta\Omega' = 2\pi(\cos\theta'_1 - \cos\theta'_2)$ sr, where $\theta'_1 = \theta - \Delta\theta'/2$ and $\theta'_2 = \theta + \Delta\theta'/2$. The linearity is testified by a weak deviation from the unity ratio given below

$$R = \frac{N'_{e+}}{\Delta\Omega'} \Big/ \frac{N_{e+}}{\Delta\Omega}. \tag{8}$$

Here $N'_{e+}/\Delta\Omega'$ is the yield of positrons N'_{e+} in case of a larger solid angle $\Delta\Omega'$, normalized to this angle, $N_{e+}/\Delta\Omega$ is the same for the smaller solid angle. In practical terms, it may be considered that if $\Delta\theta$ is between 1°and 5°, and $\Delta\theta' \geq 2\Delta\theta$, then the deviation from the unity ratio R roughly corresponds to the relative error in the calculated yield N'_{e+} value,

which arises due to the increase in the solid angle from $\Delta\Omega$ to $\Delta\Omega'$. The admissible value of the error depends on the (e^+, e^-)-pair background contribution to the electron spectrum. In our yield calculations at $\theta = 60°$, the angular acceptance of the spectrometer specified the angle $\Delta\theta = 4.8°$, and for the computation we took $\Delta\theta' = 10°$, which was consistent with $\Delta\Omega' = 0.95$ sr and $R = 1.1 \pm 0.1$.

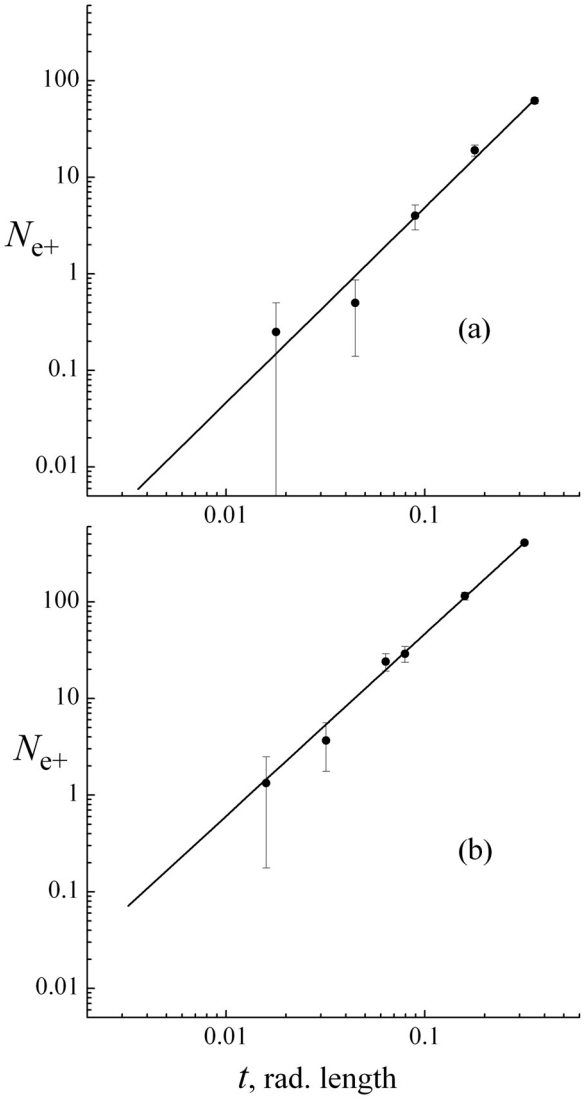

Fig. 3. Positron yields N_{e+} versus target thickness t.
a) The points show the calculation for the ^{208}Pb target at E_γ = 330 MeV and $\theta = 60°$, the solid line is the result of function (9) fitting to the points; b) the same but for the ^{6}Li target at E_γ = 250 MeV and $\theta = 15°$.

For all that, the application of the above-given technique shows that the PC operation for a few hours ensures the computation of positron yield from the ^{208}Pb target within a statistical uncertainty of about 10%, but only for $t \geq 0.1$ rad. length, while the real target thickness t_0 is of about 0.01 rad. length. One can try to calculate the positron yield from such thin targets by carrying out the following operating sequence:

a) calculation of the yield N_{e+} for a set of values of $t \geq 0.1$ rad. length;
b) the finding of the function $N_{e+}(t)$,which adequately describes these calculated data;
c) extrapolation of the function $N_{e+}(t)$ to the region of $t < 0.1$ rad. length, i.e., $t = t_0$.

The success of applying the described procedure depends on the choice of the function $N_{e+}(t)$. Let us examine the function.

If in the passage of the photon through the substance only the reaction of (e^+, e^-)-pair photoproduction took place, then, as it follows from the definition of the term "reaction cross-section", the positron yield would be linearly dependent on the target thickness: $N_{e+,1} \propto t$. To arrive at the spectrometer, the as-produced positron should be scattered through the angle $\theta \pm \Delta\theta/2$, which results from positron interaction with the target substance. If this interaction is described only by the cross-section for scattering by the electron, then the positron yield should also be linearly dependent on the target thickness $N_{e+,2} \propto t$. Since both the photoproduction and scattering reactions take place in the target B in sequence, then the yield for the positron coming to the spectrometer can be written as $N_{e+} \propto t^2$. At small target thicknesses, the large-angle positron trajectory deviation occurs mainly due to positron-electron scattering. However, at these conditions other, less probable, processes manifest themselves. Therefore, in the t-dependence of N_{e+} under consideration, the exponent on t is not exactly equal to 2. The GEANT-3 program can calculate the processes that exert influence on the positron motion in the substance. Using this program, we have calculated a variety of positron yields in a wide range of t values for the following cases: ^{208}Pb target at E_γ = 330 and 610 MeV, exit angle $\theta = 60°$; ^{208}Pb target at E_γ = 330 MeV, $\theta = 20°$; and also, ^{6}Li target at E_γ = 250 MeV, $\theta = 15°$ and 35°.

The calculation results relating to each of the energies E_γ were reduced to the same number of photons $n_{\gamma,0}$, which corresponds, according to the DT model, to the photon yield from the target A, as n_{e-} electrons are incident on it. Then the calculated positron yields were fitted with the function

$$N_{e+}(t) = A \times t^p \tag{9}$$

where A and p are the variable parameters (Fig. 3). The powers resulting from fitting were as follows: p = 2.0 to 2.1 for the lead target; 1.6 to 1.9 for the lithium target. On substitution of the target A thickness t_0 into function (9) with the obtained parameters, we get the positron yield from the target under consideration. We denote this yield by $N_{e+,m}(t_0)$, and assume that the $N_{e+,m}(t_0)$ value, derived in this way, has been determined with a reasonable degree of accuracy. However, it took more than 24 hours of PC operating time to compute the positron yield, which corresponds to one

photon energy value. This time consumption is admissible in the elaboration of the computation procedure, but is undesirable in actual practice.

A further computational shortcut consisted in choosing the optimum set of the yields to be computed. So, from the array of N_{e+} for a wide range of t values employed to obtain the yield $N_{e+,m}(t_0)$, different sets of these values were chosen, and function (9) was fitted to them. Each fitting gave its $N_{e+,j}(t_0)$, where the index j denoted the set of computed yields used in the fitting. As a result, the group of $N_{e+,j}(t_0)$ values was formed, from which the nearest to $N_{e+,m}(t_0)$ were chosen. The computations were carried out at different photon energies and angles of scattering by ^{208}Pb and ^{6}Li targets. As a result, the most optimum set was found to consist of two yields corresponding to the target thicknesses $t \approx 0.08$ and 0.33 rad. length. The $N_{e+,j}(t_0)$ values computed on the basis of these yields differed from the $N_{e+,m}(t_0)$ values by no more than 10% or 15%. This difference specifies the limit of accuracy of further computations, where the computed positron yields corresponding to the mentioned thicknesses are used to obtain the yield $N_{e+}(t_0)$.

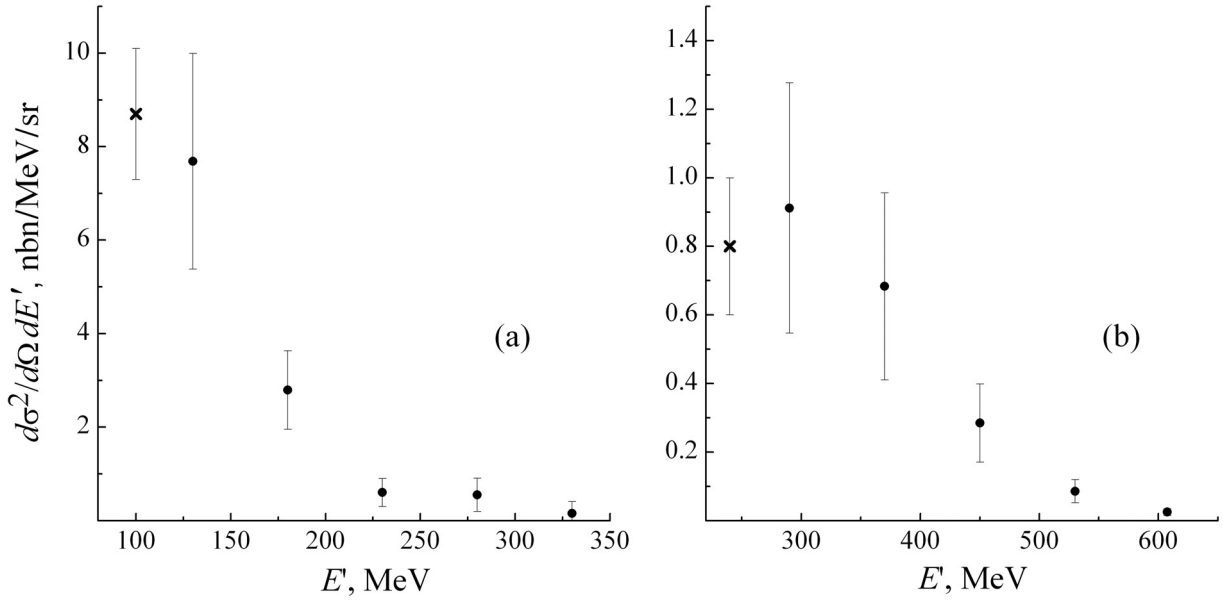

Fig. 4. Positron spectrum

a) ^{208}Pb target, 1.6×10^{-2} rad. lengths thick, at $E_0 = 354$ MeV and $\theta = 60°$. The points show the present calculated data; the cross shows the experimental value taken from [3]; b) the same but for the positron spectrum corresponding to $E_0 = 645$ MeV.

Using this method, we calculate the positron yield from the ^{208}Pb target at $\theta = 60°$ for photons of energies 130, 180, 230, 280, 330 MeV. Then, by means of (7) we obtain the positron spectrum corresponding to the electrons of energy $E_0 = 354$ MeV incident on the target. As stated above, a similar spectrum was obtained by experiment in work [3]. From the figure given there[1], the yield of positrons of energy $E' = 100$ MeV can be estimated to be 8.7 ± 1.4 nbn MeV^{-1} sr^{-1}, this being consistent with our calculated data (Fig. 4a). The authors of paper [3] also gave the experimental positron spectrum from the ^{208}Pb target measured at $E_0 = 645$ MeV, $\theta = 60°$. Our positron spectrum calculations carried out for this case are also in agreement with the experimental result (Fig. 4b).

In the experimental works [4] and [5] the important results of the Coulomb Sum values of ^{6}Li and ^{7}Li respectively were obtained, however the detailed measurements of positrons emission from the target were not carried out and so the background of (e^+, e^-)-pairs wasn't taken account. Using the developed method we calculated the positrons spectra for typical measurement conditions [4,5]: $E_0 = 260$ MeV, $\theta = 35°$ and $E_0 = 135$ MeV, $\theta = 160°$. According to the performed calculations, the maximum value of ratio the (e^+, e^-)-pairs background to the scattered electrons yield was 4×10^{-4} in the case of $\theta = 35°$, and was 5×10^{-3} in the case of measurements under $\theta = 160°$. These obtained values correspond to the tails of quasy-elastic electron scattering peaks, where the experimental errors amount to 20% ÷ 50%. Thus the neglecting of the background of (e^+, e^-)-pairs in works [4] and [5] may be consider as justified.

[1] For ease of comparison between our present calculations and the data of [3], Fig. 4 shows the (e^+, e^-)-par background in the same form [3], i.e., as a twice differential cross-section. It should be noted that this representation of the (e^+, e^-)-par background is incorrect. That is, in view of the $e^- \rightarrow (e^{-\prime}, \gamma)$ reaction taking place in the target A, the yield of the (e^+, e^-)-par background is $N_{e+} \propto t^3$. However, according to the definition of the reaction "cross-section", the reaction yield is directly proportional to the target thickness. Therefore, what is shown here in Fig. 4 and in Figs. of [3] may be called "the pseudo cross-section" corresponding to the target thickness t". If comparison is made between two pseudo cross-sections corresponding to the same target thickness (Fig. 4), or between the reaction cross-section and the background cross-section [3] (both cross-sections being measured on the same target), then in both cases this is merely the comparison between two yields represented in not exactly correct form.

CONCLUSION

The main result of the present paper lies in the development of the methods, which in the case of electronuclear experiment enable one to compute (with a PC) the background of (e^+, e^-)-pair photoproduction on both heavy- and light- nuclear targets (^{208}Pb and ^{6}Li, respectively). In each case, it took about 12 hours for the PC to compute the positron spectra by these methods (Fig. 4a, b), which may be considered quite acceptable for practical work.

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